

Exercise 40

Evaluate the integral.

$$\int_1^3 \frac{y^3 - 2y^2 - y}{y^2} dy$$

Solution

Split up the fraction into three terms. Then split up the integral by using the properties of integrals in order to integrate each term separately.

$$\begin{aligned} \int_1^3 \frac{y^3 - 2y^2 - y}{y^2} dy &= \int_1^3 \left(\frac{y^3}{y^2} - \frac{2y^2}{y^2} - \frac{y}{y^2} \right) dy \\ &= \int_1^3 \left(y - 2 - \frac{1}{y} \right) dy \\ &= \int_1^3 y \, dy - \int_1^3 2 \, dy - \int_1^3 \frac{1}{y} \, dy \\ &= \left(\frac{y^2}{2} \right) \Big|_1^3 - (2y) \Big|_1^3 - (\ln y) \Big|_1^3 \\ &= \left(\frac{3^2}{2} - \frac{1^2}{2} \right) - [2(3) - 2(1)] - (\ln 3 - \ln 1) \\ &= 4 - 4 - \ln 3 \\ &= -\ln 3 \end{aligned}$$